A Unique Key Method for Efficient Image Retrieval in Symbolic Image Databases

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***Abstract*[+]—As the number of images in the database increases tremendously, efficient retrieval of images with a desired content from a symbolic image database becomes a challenging and motivating research issue. In this paper, to efficiently support both of exact and similarity retrieval in symbolic image databases, we propose the UniKey method, which maps each quadruple into a unique key and could reduce the needed storage space up to 3/4.**

***Keywords*—databases, hash function, iconic index, similarity retrieval, symbolic image**

I. Introduction

An image database plays an important role in many applications, including geographical information systems, computer aided design, and medical image archiving. The method of representing images is one of the major concerns in designing an image database system [3]. One way of representing an image is to construct a *symbolic image* for that image [9]. Instead of the low pixel level features, a symbolic image representation consists of the high-level features for the given image, such as pairwise spatial relationships between objects, which well capture the knowledge about the content of the given image [8]. However, as the number of images in the database increases tremendously, efficient retrieval of images with a desired content from a symbolic image database becomes a challenging and motivating research issue [4][5].

Basically, there are two types of retrieval [5][6]: exact match retrieval and similarity retrieval. Given a query symbolic image, exact match retrieval retrieves identical images from the database, while similarity retrieval retrieves all images similar to the query image according to some similarity measures. There have been many methods proposed for the symbolic image representation and retrieval. The 9D-SPA method [4] uses quadruples to represent the spatial relationship for any two objects in one image, which needs large storage space. Punitha and Guru's method [5] uses triples to represent the spatial relationships, and computes a unique key for each triple. Although this method reduces the needed storage space, it could not support the similarity retrieval.

To reduce the needed storage space and support similarity retrieval, in this paper, we propose the *UniKey* (Unique Key) method. We make use of the 13 spatial relationships proposed in [1] to represent a symbolic image in the form of quadruples. This representation is well adaptable to similarity retrieval, since it needs only the relative spatial relationships between objects. Moreover, we propose a new hash function to map each quadruple into a unique key, which could reduce the needed storage space up to 3/4. We also prove that there is no ambiguity between each quadruple and its mapping key. Therefore, we will not obtain any undesired quadruple with such a mapping. Furthermore, we propose a hash table and a new similarity measure for supporting both of exact match retrieval and similarity retrieval. In our analysis, we study the time complexity of the UniKey method.

The rest of this paper is organized as follows. In Section II, we introduce the 13 spatial relationships between two objects. Section III presents the proposed UniKey method. In Section IV, we analyze the time complexity of the UniKey method. Finally, we make a conclusion in Section V.

II. Background

The fundamental idea of the symbolic projection is to project the objects of an image along the *x* and *y*-directions to obtain the relative positions of objects in the *x* and *y*-axis, respectively. In this paper, we apply the 13 spatial relationships [1][2] shown in Figure 1 to form the quadruples for a symbolic image. For two objects O_i and O_j in an image, the corresponding quadruple is denoted as $(O_i, O_j, X_{ij}, Y_{ij})$, where X_{ij} and Y_{ij} are identifiers of the spatial relationships between objects O_i and O_j along the *x* and *y*-directions, respectively.

III. The UniKey Method

In this section, we present the equations for computing the unique key of a quadruple. We also prove that the keys computed by our equations cause no ambiguity, *i.e.*, one unique key per quadruple. Then, based on these equations, we present a hash table and a new similarity measure for efficiently supporting both of exact and similarity retrieval in symbolic image databases.

First, we describe the equation proposed in [4] for transforming pair (O_i, O_j) into one unique value, U_{ij}.

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Although this equation has been proposed in [4], they did not prove that this equation has a one-to-one mapping

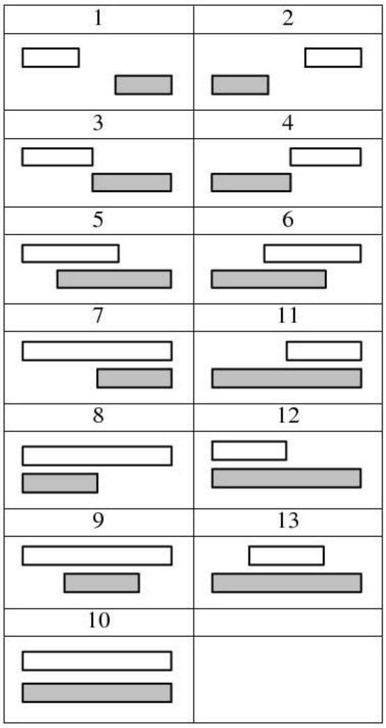

Figure 1. The identifiers of 13 spatial relationships between two objects in one dimension (horizontal projection)

property. In Theorem 1, we will further prove the one-to-one mapping property of this equation.

Theorem 1. Suppose there are two objects, O_i and O_j, where i and j are positive integers, and $i < j$. U_{ij} is calculated by Equation 1. Then, there is a one-to-one mapping from the pair (O_i, O_j) to U_{ij}.

$$U_{ij}=\frac{(j\text{-}1)(j\text{-}2)}{2}+i \qquad (1)$$

Proof of Theorem 1. We prove the following two lemmas to conclude that Equation1 is a one-to-one mapping.

Lemma 1. $U_{ij} < U_{kj}$, if $i < k$.

Proof of Lemma 1. $U_{kj} - U_{ij} =$

$$\left[\frac{(j-1)(j-2)}{2}+k\right]-\left[\frac{(j-1)(j-2)}{2}+i\right]=k-i>0$$

Lemma 2. $U_{(j\text{-}1)j} < U_{1k}$, if $j < k$.

Proof of Lemma 2. Since (j-1) and j are positive integers, we have $2 \le j < k$. Let $j = a + 2$ and $k = a + 2 + b$, where $a \ge 0$ and $b \ge 1$. Then, $U_{1k} - U_{(j-1)j}$

$$=\left[\frac{(k-1)(k-2)}{2}+1\right]-\left[\frac{(j-1)(j-2)}{2}+(j-1)\right]$$

$$= b^2 + b + a \times (2b-1) > 0$$

From Lemmas 1 and 2, we conclude that for any two U_{ij} and $U_{i'j'}$, if $i \neq i'$ or $j \neq j'$, there will exist an order between them; that is, there exists a one-to-one mapping from pair (O_i, O_j) to U_{ij}, since $U_{i'j'}$ mapped by another pair ($O_{i'}$, $O_{j'}$) is either greater than or less than U_{ij}. □

Next, in Theorem 2, we describe the equation for transforming a quadruple into a unique value, and prove that this equation also has a one-to-one mapping property.

Theorem 2. Suppose there are two objects, *e.g.*, O_i and O_j. The spatial relationships between O_i and O_j along x and y-directions are X_{ij} and Y_{ij}, respectively, where $1 \le X_{ij}, Y_{ij} \le 13$. U_{ij} is calculated according to Equation 1. Then, given $m > 13$ and $R > U_{ij}$, Equation 2 is a one-to-one mapping to function $\boldsymbol{h}$.

$$h(U_{ij}, X_{ij}, Y_{ij}) = (R \times X_{ij} \times \mathrm{m}) + (R \times Y_{ij}) + U_{ij} \qquad (2)$$

Proof. Assume that there are two distinct triples, *e.g.*, (U_{ab}, X_{ab}, Y_{ab}) and (U_{cd}, X_{cd}, Y_{cd}), and their values of function h are the same, *i.e.*, $h(U_{ab}, X_{ab}, Y_{ab}) = h(U_{cd}, X_{cd}, Y_{cd})$. Thus, we have

$$(\mathrm{R} \times X_{ab} \times m) + (R \times Y_{ab}) + U_{ab}$$

$$= (R \times X_{cd} \times m) + (R \times Y_{cd}) + U_{cd}$$

$$\Rightarrow R \times (X_{ab} - X_{cd}) \times m + R \times (Y_{ab} - Y_{cd}) + (U_{ab} - U_{cd}) = 0 \qquad (2a)$$

Let $x = X_{ab} - X_{cd}$, $y = Y_{ab} - Y_{cd}$, and $u = U_{ab} - U_{cd}$. Then, Equation 2a can be written as follows.

$$R \times x \times m + R \times y + u = 0$$

$$\Rightarrow R \times (x \times m + y) = -u \qquad (2b)$$

We conclude that u is a multiple of R. Because U_{ab} is calculated according to Equation 1, we have

$$1 \le U_{ab} < R \Rightarrow (1 - U_{cd}) \le (U_{ab} - U_{cd}) < (R - U_{cd})$$

$$\Rightarrow (1 - R) < (1 - U_{cd}) \le (U_{ab} - U_{cd}) < (R - U_{cd}) < (R - 1)$$

$$\Rightarrow (1 - R) < (U_{ab} - U_{cd}) < (R - 1)$$

$$\Rightarrow |U_{ab} - U_{cd}| < |R - 1|$$

$$\Rightarrow |U_{ab} - U_{cd}| < |R - 1| < |R|$$

$$\Rightarrow |U_{ab} - U_{cd}| < |R|$$

$$\Rightarrow |u| < |R|$$

Because u is a multiple of R and $|u| < |R|$, u must be zero. This implies $U_{ab} = U_{cd}$. Then, from Equation 2b, we have

$$R \times (x \times m + y) = 0 \Rightarrow x \times m + y = 0$$

$$\Rightarrow x \times m = -y \qquad (2c)$$

We conclude that y is a multiple of m. Because $1 \le Y_{ab}, Y_{cd} \le 13$ and $13 < m$, we have

$$1 \le Y_{ab} \le m \Rightarrow (1 - Y_{cd}) \le (Y_{ab} - Y_{cd}) < (m - Y_{cd})$$

$$\Rightarrow (1 - m) < (1 - Y_{cd}) \le (Y_{ab} - Y_{cd}) < (m - Y_{cd}) < (m - 1)$$

$$\Rightarrow (1 - m) < (Y_{ab} - Y_{cd}) < (m - 1)$$

$$\Rightarrow |Y_{ab} - Y_{cd}| < |m - 1|$$

$$\Rightarrow |Y_{ab} - Y_{cd}| < |m - 1| < |m|$$

$$\Rightarrow |Y_{ab} - Y_{cd}| < |m|$$

$$\Rightarrow |y| < |m|$$

Because y is a multiple of m and $|y| < |m|$, y must be zero. This implies $Y_{ab} = Y_{cd}$. Then, from Equation 2c, we have

$$x \times m = 0 \Rightarrow x = 0 \Rightarrow X_{ab} - X_{cd} \Rightarrow X_{ab} = X_{cd}$$

This contradicts our assumption; that is, (U_{ab}, X_{ab}, Y_{ab}) and (U_{cd}, X_{cd}, Y_{cd}) are two distinct triples. Hence the proof. □

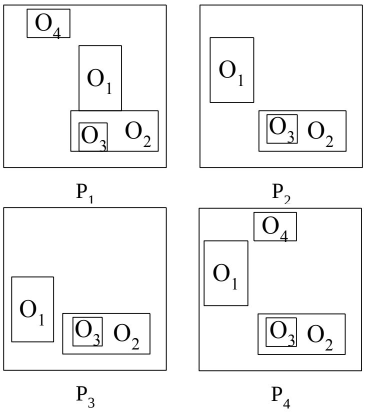

Figure 2. An example of database images

For any two objects O_i and O_j with the spatial relationships X_{ij} and Y_{ij}, we can map the quadruple (O_i, O_j, X_{ij}, Y_{ij}) to a unique value by Equations 1 and 2. Then, we can build a hash table for images in the database to make the task of image retrieval efficient. For example, suppose there is a database containing four images, identified by P_1, P_2, P_3, and P_4, as shown in Figure 2. In terms of quadruples, image P_1 could be represented as (O_1, O_2, 13, 4), (O_1, O_3, 9, 2), (O_1, O_4, 2, 1), (O_2, O_3, 9, 8), (O_2, O_4, 4, 1), (O_3, O_4, 2, 1). Since the maximal value of U_{ij} is $U_{34} = (4-1) \times (4-2)/2 + 3 = 6$ according to Equation 1, we let R be 7 (> 6). We also let m be 14. Then, we build a hash table for these four images based on Equations 1 and 2, as shown in Figure 3. For example, the hash key for the first quadruple of P_1, *i.e.*, (O_1, O_2, 13, 4), is $(7 \times 13 \times 14) + (7 \times 4) + 1 = 1303$ (Equation 2). Therefore, the identifier of image P_1 is recorded at the corresponding entry for hash key 1303. Assuming that there exists n objects in the database, the least needed size of this hash table is $R \times 13 \times m + R \times 13 + U_{(n-1)n} = 98 \times n^2 - 98 \times n + 195$.

Similarity retrieval is to measure the similarity between the query image and the database image, and then to retrieve relevant images from the database [4]. Because users may not remember the exact spatial relationships among the objects in a desired image, a set of coarse-to-fine similarity measures is also acquired. Our similarity measure is defined by Equation 3. Table 1 shows the corresponding notations used in our similarity measure.

$$Sim(Q,p) = \frac{\sum_{i=1}^{|Q|} s(p, HT[h(q_i)])}{|Q|},$$

$$\text{where } s(a,B) = \begin{cases} 1, \text{if } a \in B \\ 0, \text{otherwise.} \end{cases} \quad (3)$$

Following the previous example, assume that the set of quadruples of the query image, Q, is the same as that of image P_4, which contains 6 quadruples. The corresponding hash entries for quadruples of query Q are those denoted by symbol "*" in Figure 3. Then, since image P_4 occurs at each of these entries, the similarity measure between query Q and image P_4 is calculated as $Sim(Q, P_4)$ = (1+1+1+1+1+1) / 6 = 1, which means that image P_4 is similar to query Q with a degree of similarity 1, *i.e.*, an exact match. Similarly, the similarity measure between query Q and image P_2 is calculated as $Sim(Q, P_2)$ = (1+1+0+0+1+0) / 6 = 0.5, which means that image P_2 is similar to query Q with a degree of similarity 0.5.

key	list of image identifiers
...	...
* 113	P_2, P_4
* 114	P_2, P_4
...	...
* 123	P_4
...	...
141	P_3
142	P_3
...	...
207	P_1
208	...
209	P_1
...	...
404	P_1
...	...
* 600	P_4
...	...
898	P_1
...	...
941	P_1
...	...
* 948	P_2, P_3, P_4
...	...
* 1287	P_4
...	...
1303	P_1
...	...

Figure 3. The hash table data structure corresponding to images P_1, P_2, P_3, and P_4

Table 1. Notations used in the similarity measure

Notation	**Meaning**
Q	the set of quadruples for the given query image
$\|Q\|$	the number of quadruples in Q
q_i	the i-th quadruple in Q
$h(q_i)$	the corresponding hash key for q_i
p	an image identifier
HT	the proposed hash table
$HT[h]$	a set of image identifiers recorded at the entry of HT corresponding to hash key h

IV. Analysis

Based on the UniKey method, for each quadruple, the time complexity of the task of retrieval is O(1). If the query contains k quadruples, the time complexity is O(k). For n objects, the possible number of quadruples, *i.e.*, the number of possible combinations of any two objects, is C_2^n. Therefore, the time complexity of the task of retrieval based on our hash table is O(n^2), which is the same as that of those previous hash-oriented methods [7][10]. The major contribution of the UniKey method is the hash function for computing a unique key of a quadruple. With such a hash function, the UniKey method reduces the storage requirement up to 3/4 as compared to the 9D-SPA method [4], while it could still support similarity retrieval in symbolic image databases.

V. CONCLUSION

Efficient retrieval of images with a desired content from a symbolic image database is an important research issue. In this paper, we have proposed the UniKey method for the symbolic image representation and both of exact match retrieval and similarity retrieval. The UniKey method utilizes a new hash function to map each quadruple into a unique key, which could reduce the storage requirement. We have also proved that the keys computed by the hash function cause no ambiguity, which guarantees us against all undesired quadruples.

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